DPconv: Super-Polynomially Faster Join Ordering

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Data Systems Lab @UTN

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UTN Technische Universität Nürnberg

Join Ordering Recap

SQL Query

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SELECT *
FROM R1, R2, R3, R4
WHERE R1.a = R2.b
AND R2.c = R3.d
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Join Ordering Recap

SQL Query ⇨ Query Graph

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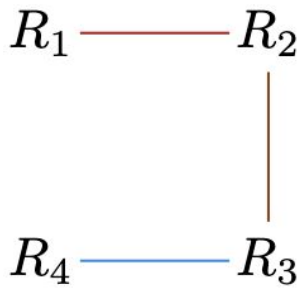

Join Ordering Recap

SQL Query ⇨ Query Graph ⇨ Query Plan

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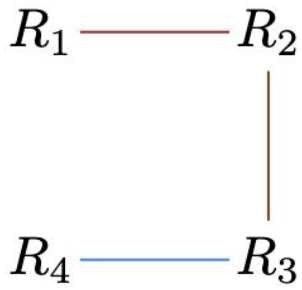

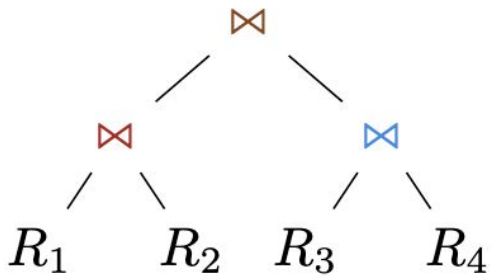

A Long Quest

SIGMOD'79 ⇒ DPsize

Access Path Selection
in a Relational Database Management System

P. Griffiths Selinger
M. M. Astrahan
D. D. Chamberlin
R. A. Lorie
T. G. Price

IBM Research Division, San Jose, California 95193

ABSTRACT: In a high level query and data manipulation language such as SQL, requests are stated non-procedurally, without reference to access paths. This paper describes how System R chooses access paths for both simple (single relation) and complex queries (such as joins), given a user specification of desired data as a boolean expression of predicates. System R is an experimental database management system developed to carry out research on the relational model of data. System R was designed and built by members of the IBM San Jose Research Laboratory.

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This paper will address the issues of access path selection for queries. Retrieval for data manipulation (UPDATE, DELETE) is treated similarly. Section 2 will describe the place of the optimizer in the processing of a SQL statement, and section 3 will describe the storage component access paths that are available on a single physically stored table. In section 4 the

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Rapid Bushy Join-order Optimization with Cartesian Products

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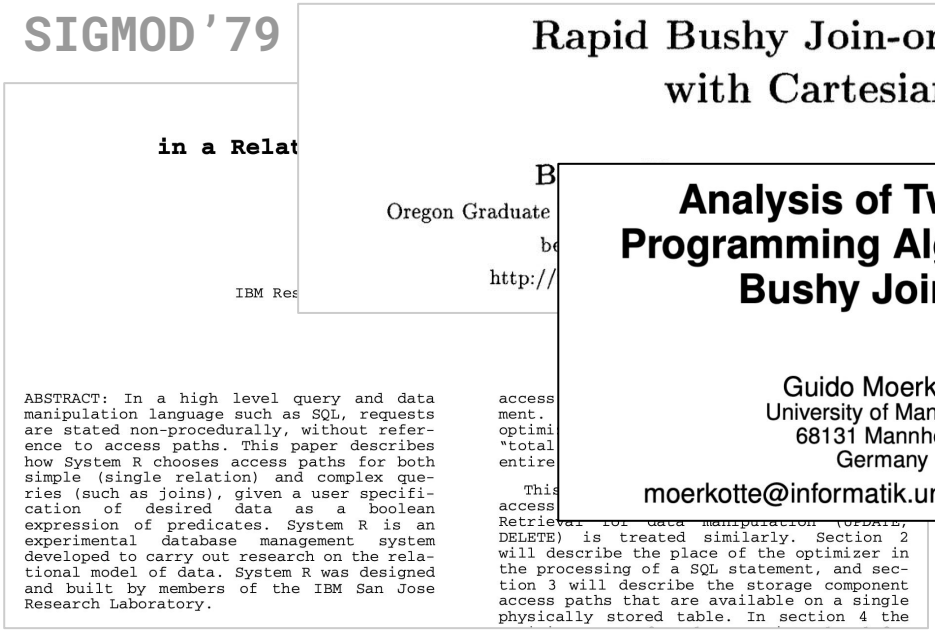
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Oregon Graduate

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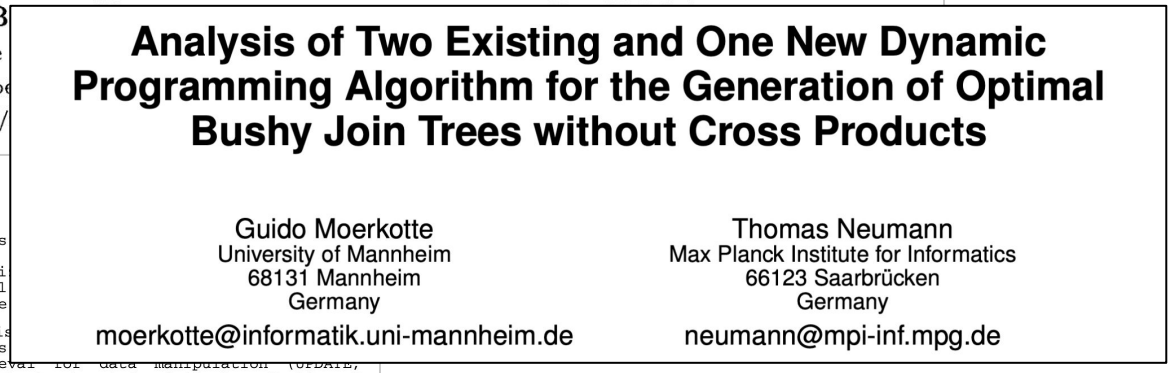
DPccp


Analysis of Two Existing and One New Dynamic Programming Algorithm for the Generation of Optimal Bushy Join Trees without Cross Products

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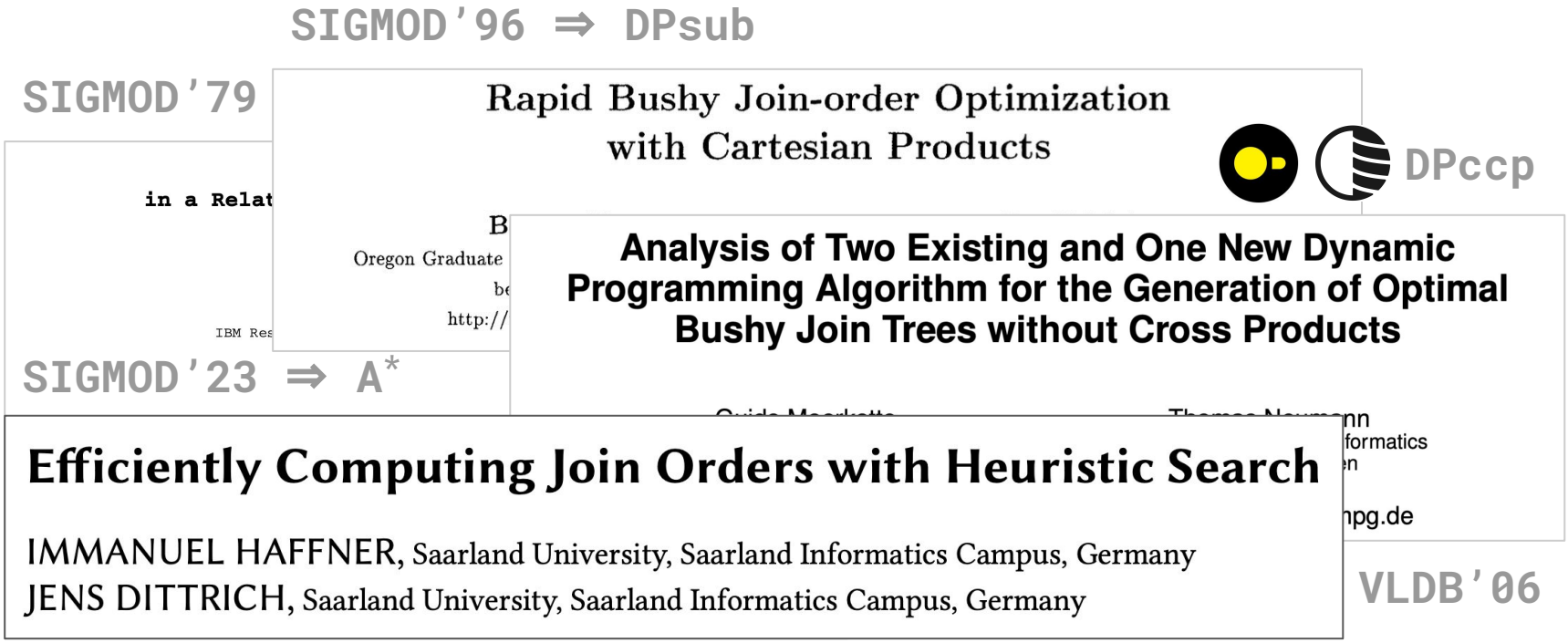
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SIGMOD'23 ⇒ A*
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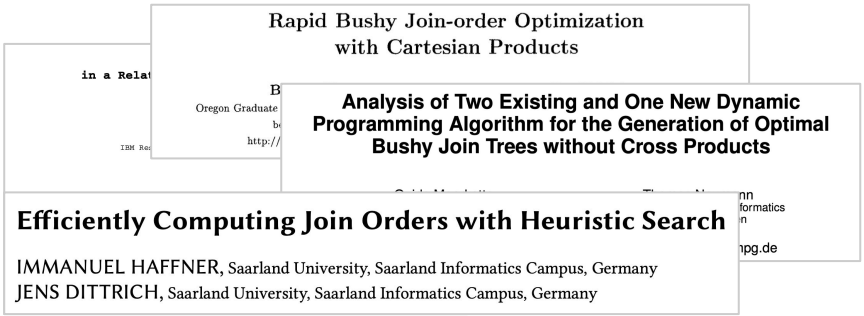
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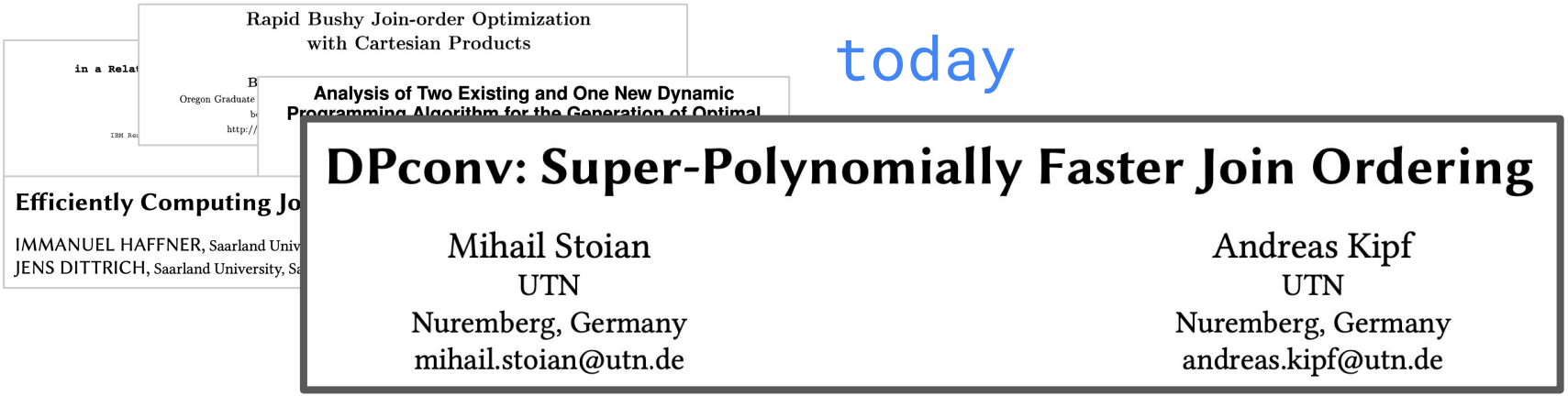
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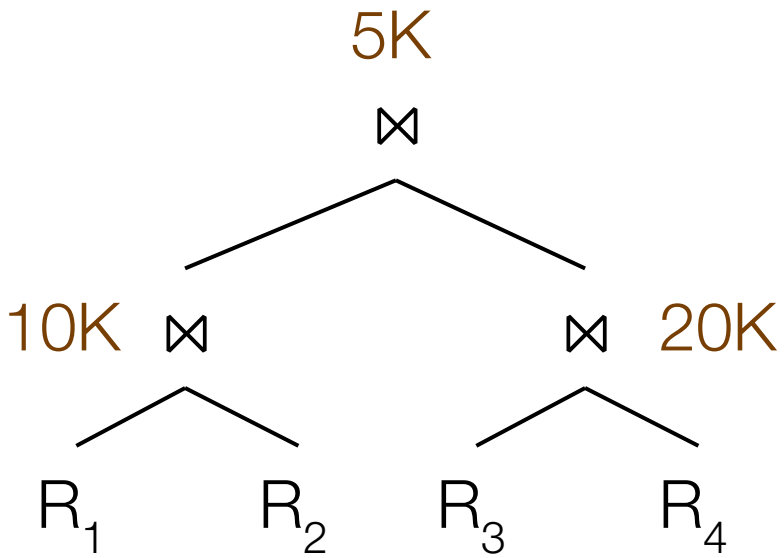
DPconv

Cost Functions



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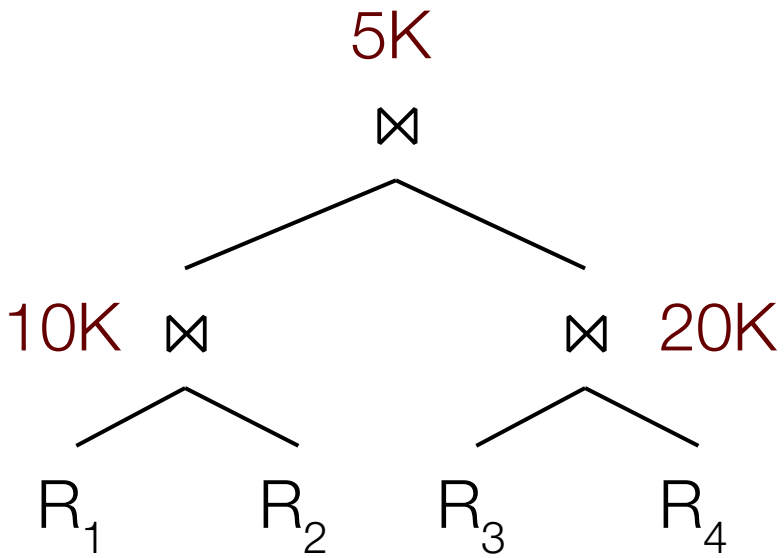

C_{out}: 10K + 20K + 5K

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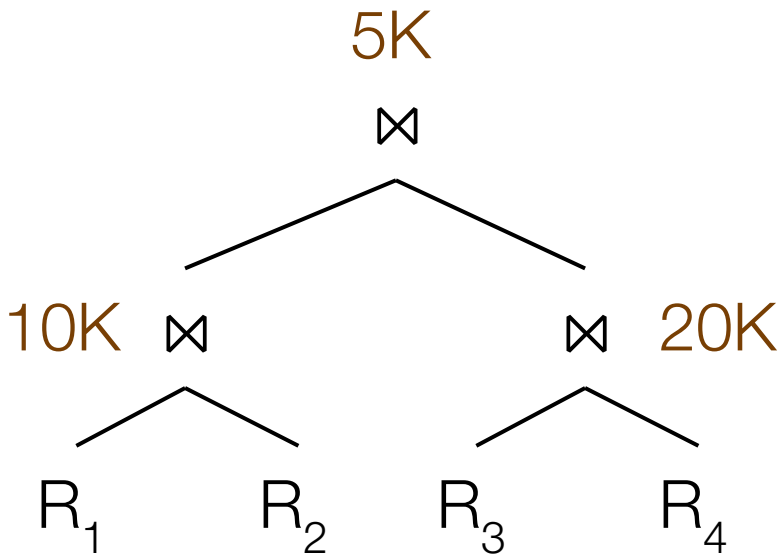

C_{out}: 10K + 20K + 5K

C_{max}: **max** {10K, 20K, 5K}

DPconv

Exact C_{max}

$O(3^n)$

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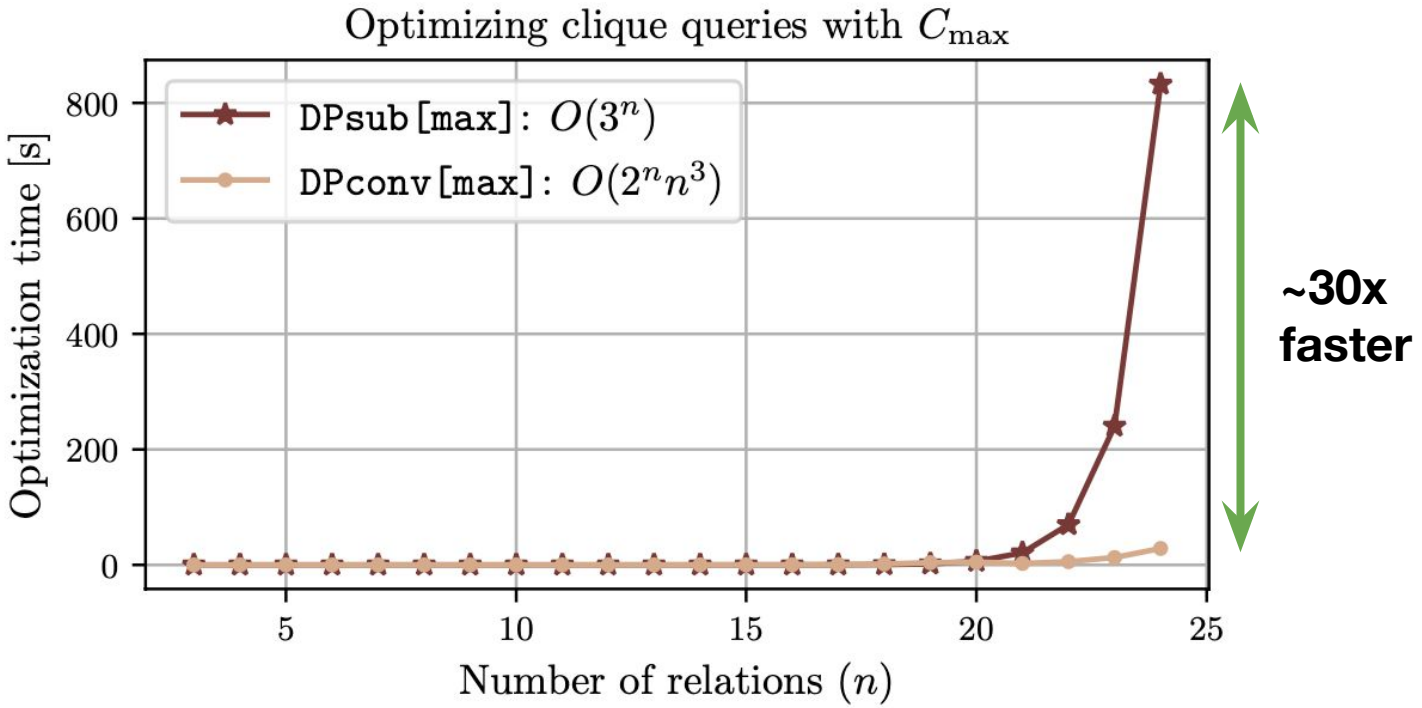
Beyond Theory



Dynamic Programming

- Bellman's optimality principle.
- Given set $S \subseteq \{1, \ldots, n\}$:
  - c(S) = Join cardinality of S.
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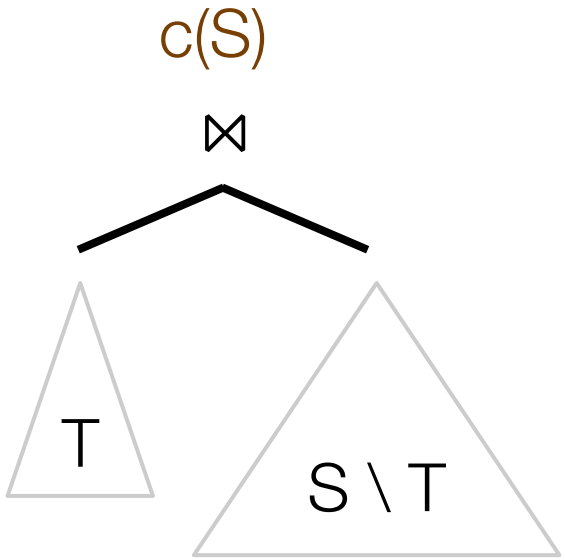

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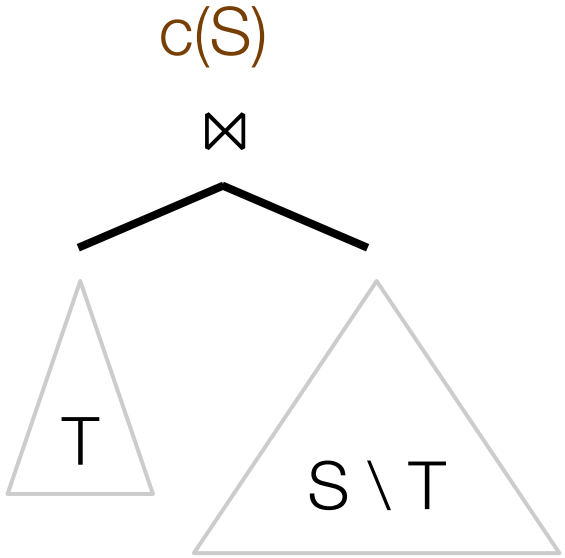

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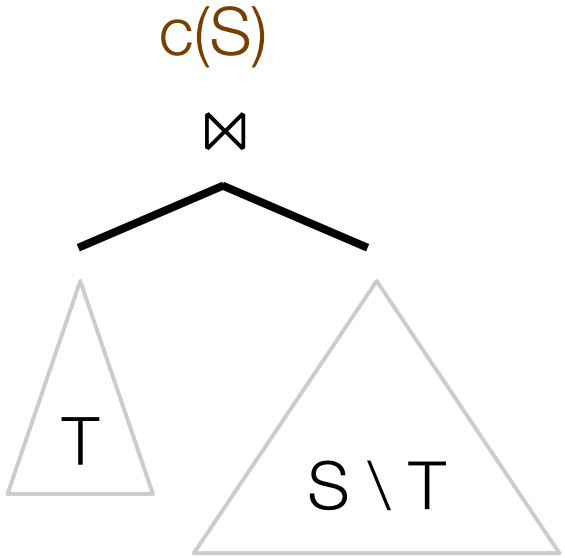

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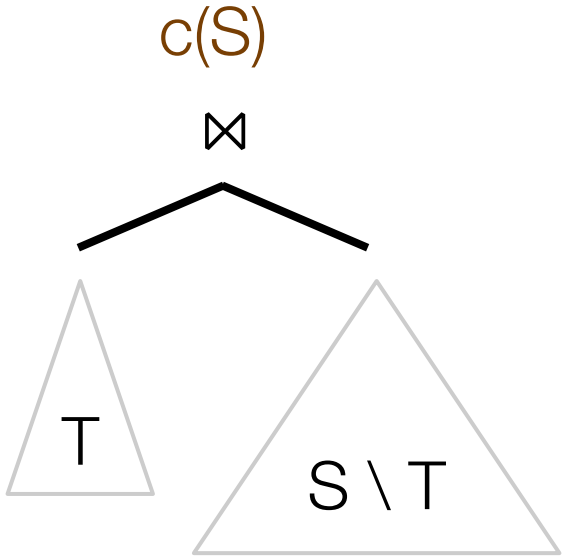

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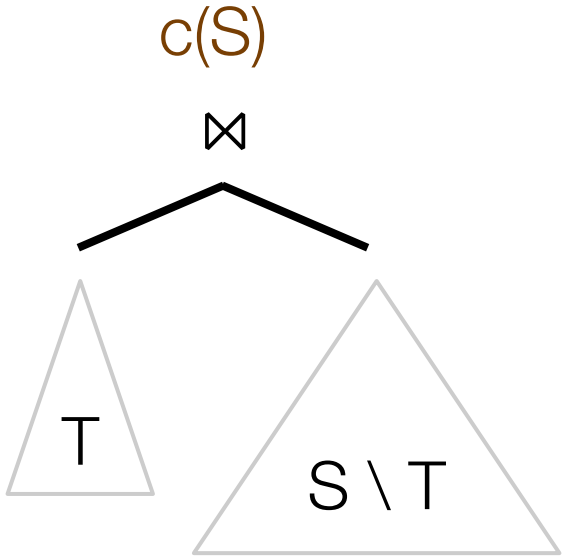

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- Running time:

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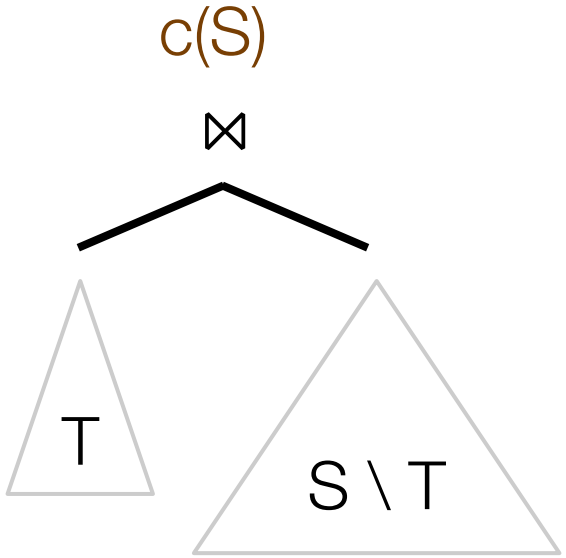

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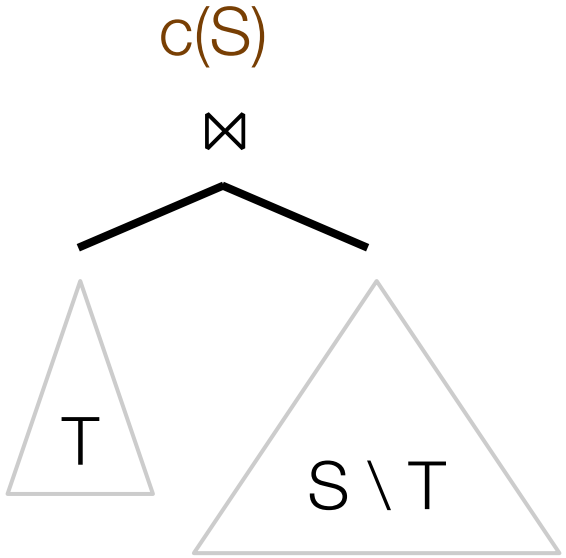

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STOC'06

FOURIER MEETS MÖBIUS: FAST SUBSET CONVOLUTION

ANDREAS BJÖRKLUND, THORE HUSFELDT, PETTERI KASKI, AND MIKKO KOIVISTO

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where addition and multiplication is carried out in an arbitrary ring. Via Möbius transform and inversion, our algorithm evaluates the subset convolution in $O(n^2 2^n)$ additions and multiplications, substantially improving upon the straightforward $O(3^n)$ algorithm. Specifically, if the input func-

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- Ring example: (**+**, ×).
  - 2 **+** 3 = 5.
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- Semi-ring example: (**min**, +), (min, max).
  - **min**(2, 3) = 3.
  - *How to reverse?*

How to Map?

- Find a mapping:

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- **Drawback**: Running time becomes *pseudo-polynomial*.
  - O(W) overhead, where W is the largest value.

Takeaway

- It is possible to map a semi-ring to a ring. **Drawback**:

$$O(2^n n^2) \Longrightarrow O^*(2^n W).$$

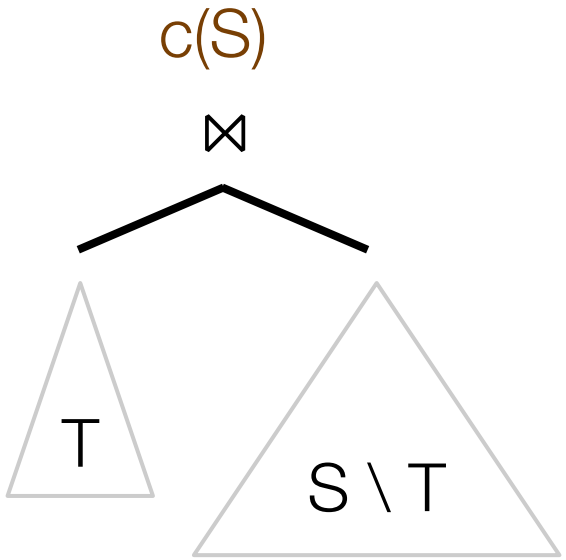
- There is a case where we can avoid the O(W) factor.

Avoiding pseudo-polynomiality

Optimizing C_{max}

- Ad-hoc dynamic programming:

$$\mathrm{DP}[S] = \max\{\mathrm{DP}[T], \mathrm{DP}[S \setminus T]\}$$



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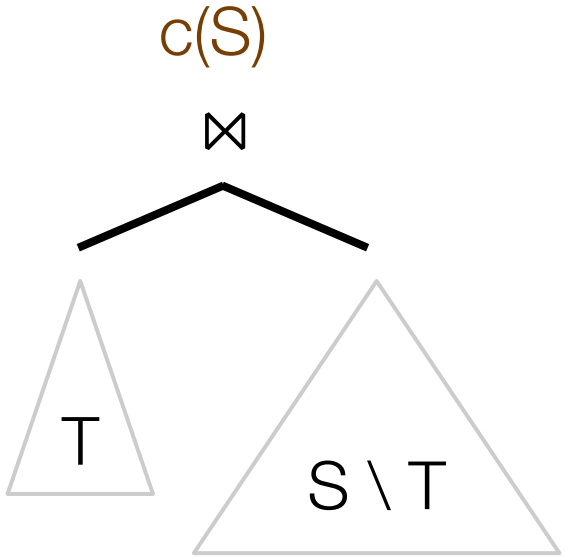

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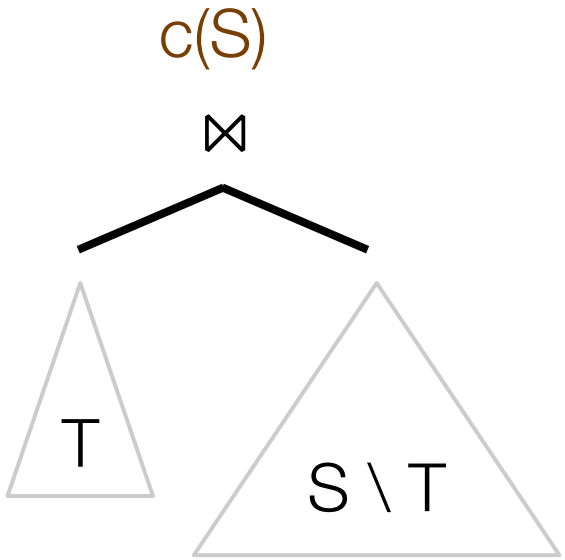

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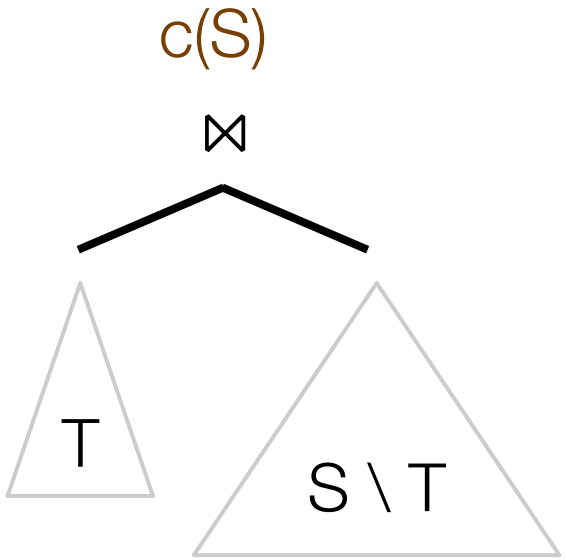
- 💡 The DP does not create new values!



Optimizing C_{max}

- Ad-hoc dynamic programming:

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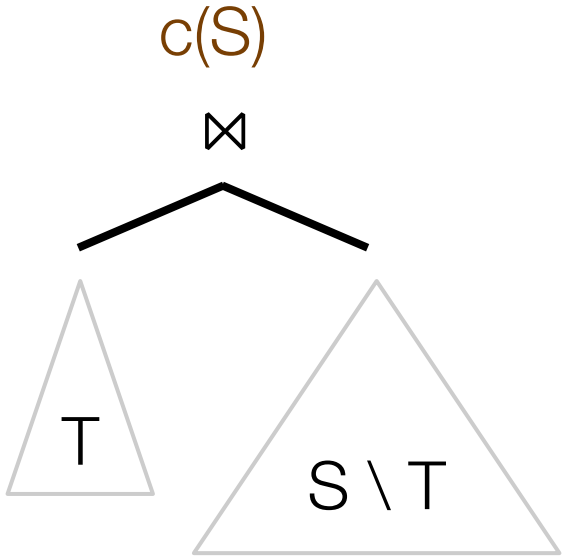
  ⇒ We can binary search DP[{1, …, n}].



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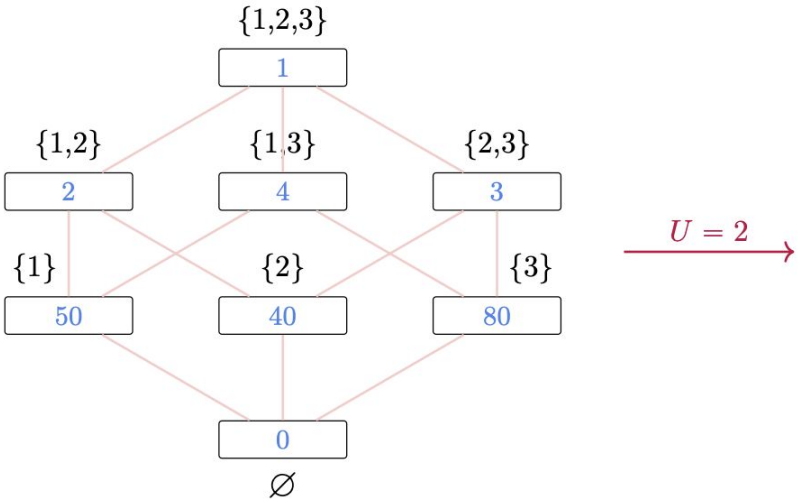

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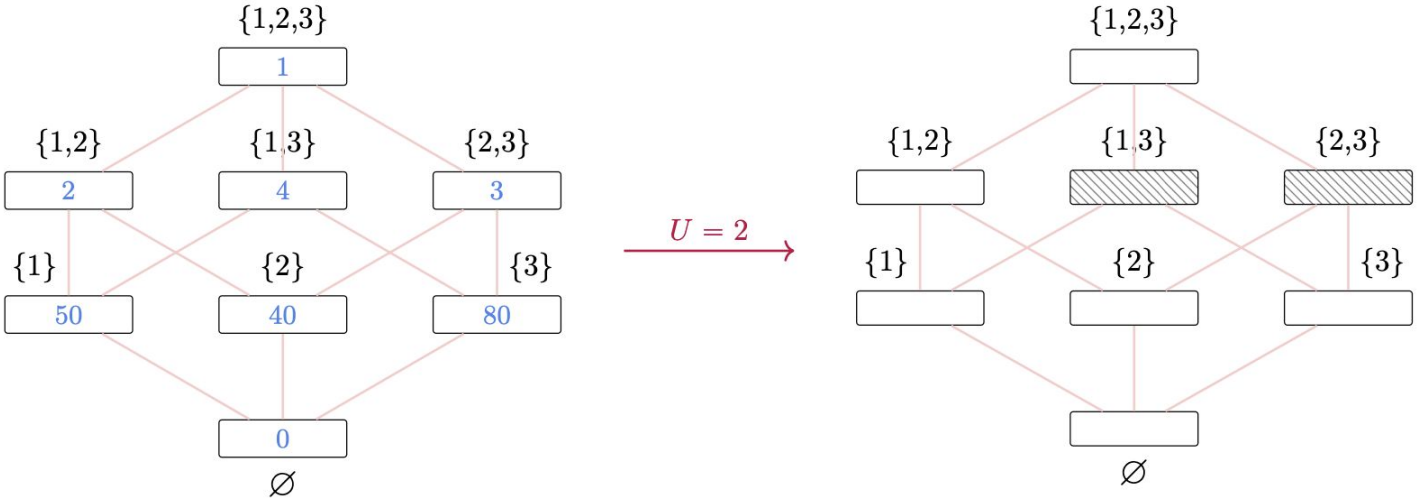

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- Running time: $O(\log(2^n) \cdot 2^n n^2) = O(2^n n^3)$.

The End?

- DPconv does not take into account the sparsity of the graph.

  ⇒ We need a sparse subset convolution.

- *Polynomial-space* join ordering?

  ⇒ Preliminary results for acyclic query graphs.