DPconv:

Super-Polynomially Faster Join Ordering

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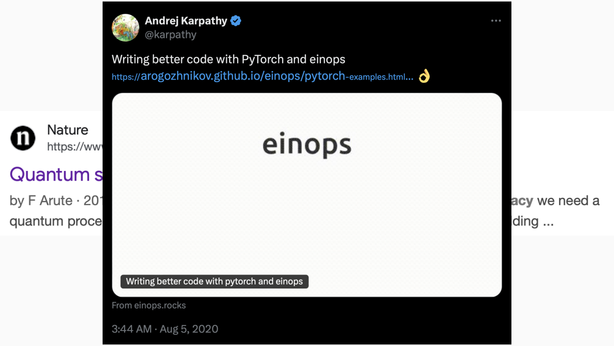
It's not just about join ordering..

Nature
https://www.nature.com › articles

Quantum supremacy using a programmable ...

by F Arute · 2019 · Cited by 9186 — Therefore, in order to claim **quantum supremacy** we need a quantum processor that executes the program with sufficiently low error rates. Building ...

It’s not just about join ordering..



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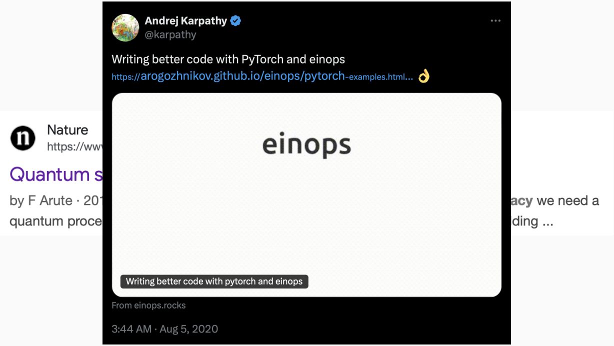

First super-polynomial speedups for `einsum` optimization.

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Problem Statement

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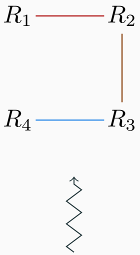
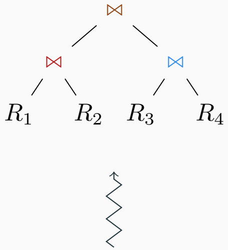
- Research question since $\sim$50 years: *How fast can we get?*

Join Ordering: Notation

SELECT *
FROM R_1, R_2, R_3, R_4
WHERE R_1.a = R_2.b
AND R_2.c = R_3.d
AND R_3.e = R_4.f



Query graph



Join tree

Join Ordering: Dynamic Programming

Dynamic Programming

- Use Bellman's optimality principle.
- Dynamic programming table $\mathrm{DP}[S]$: The optimal cost to join the relations in the set S.

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Running Time Analysis

$$\sum_{S \subseteq [n]} 2^{|S|} = \sum_{k=0}^{n} \binom{n}{k} 2^k = (1+2)^n = 3^n.$$

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Join Ordering: Status Quo

Exact Algorithms

- Selinger et al. (1979): `DPsizeLinear`[2] $\rightarrow \Theta(2^n n)$.

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Approximation Algorithms

- Many good heuristics, yet no theoretical guarantees.
- For a good reason: it's *hard* [1]:
  - NP-hard to approximate the optimal cost K within $2^{\Theta(\log^{\delta-1} K)}$ for any $\delta > 0$.

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Our Results

`TL;DR`

Join Ordering DP = Subset Convolution

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- C_{out}: First $(1+\varepsilon)$-approximation algorithm.
- Beyond databases: `einsum` optimization.
  - Used in quantum circuit simulation.
  - Speedup over the state-of-the-art algorithm in `opt_einsum`.

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General Framework

- Join ordering DP:

$$\mathrm{DP}[S] = c(S) \otimes \min_{T \subseteq S} (\mathrm{DP}[T] \otimes \mathrm{DP}[S \setminus T]).$$

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Overview: Part II

FOURIER MEETS MÖBIUS: FAST SUBSET CONVOLUTION

ANDREAS BJÖRKLUND, THORE HUSFELDT, PETTERI KASKI, AND MIKKO KOIVISTO

Abstract. We present a fast algorithm for the *subset convolution problem*: given functions f and g defined on the lattice of subsets of an n-element set N, compute their *subset convolution* $f * g$, defined for all $S \subseteq N$ by

$$(f * g)(S) = \sum_{T \subseteq S} f(T) g(S \setminus T)\,,$$

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  - Represent a value v as x^v.
  - **Intuition**:
    - "$+$" becomes product: $x^{a+b} = x^a x^b$.
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Where to go?

Overview: Part III

`DPconv`: **Takeaway**

- *No* pseudopolynomial factor when we optimize for $C_{\max}$.

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Overview: Part III

DPconv: Takeaway

- *No* pseudopolynomial factor when we optimize for $C_{\max}$.
- *No* pseudopolynomial factor when we move to $(1+\varepsilon)$-approximation $\rightarrow$ C_{out} can be $(1+\varepsilon)$-approximated in $O^*(2^n \log W/\varepsilon)$.[4]

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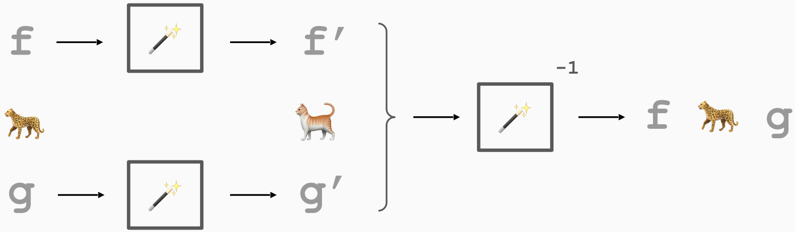
Fast Convolutions: Intuition



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- Our magic box – **zeta transform**:

$$\zeta f(S) = \sum_{T \subseteq S} f(T),$$

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Zeta Transform: Hands-On

- You have 1 minute to fill up:

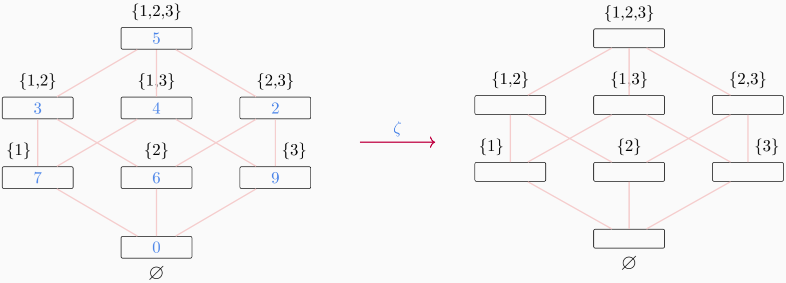

Figure 1: How to compute the zeta transform of a set function.

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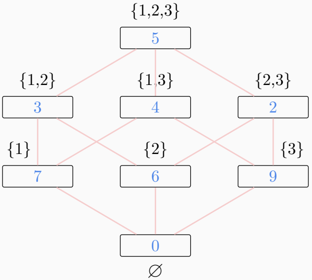
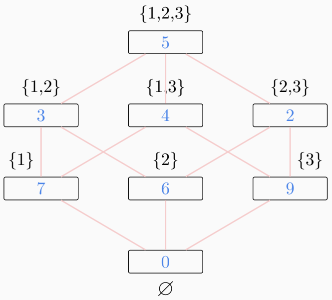


Figure 2: How to compute the zeta transform of a set function.

Fast Zeta Transform: Pseudocode

```
zeta(f):
  for d in range(n):
    for S in range(2**n):
      if S & 2**d:
        f[S] += f[S ^ 2**d]
```



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Inverse Zeta Transform: Example

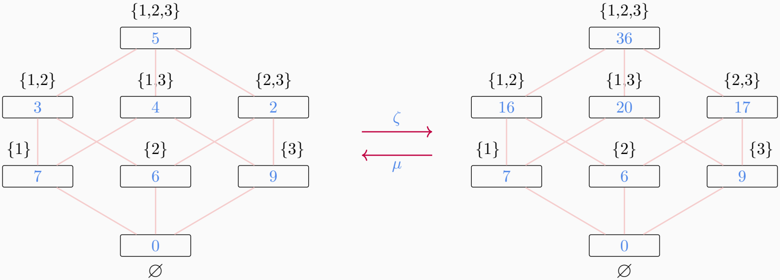

Figure 3: Inverting the zeta transform.

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How can we fix it?

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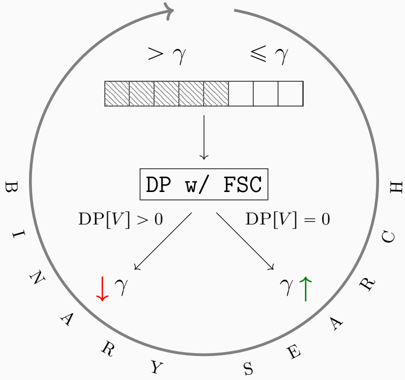

Figure 4: How `DPconv` optimizes $C_{\max}$.

Optimizing C_{max}: Example

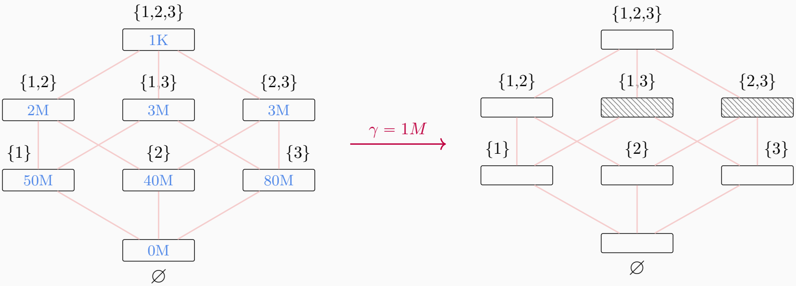

Figure 5: The cardinalities $> \gamma$ are ignored.

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  - Is the pseudopolynomial $O(W)$ factor actually needed?

Joint Optimization: $C_{\mathrm{cap}} = C_{\mathsf{max}} + C_{\mathrm{out}}$

Motivation C_{cap}

- Optimizing for C_{max} alone may lead to slow plans.
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- First run `DPccp` with $C_{\max}$, then C_{out} with a threshold.
- However, this is slower:

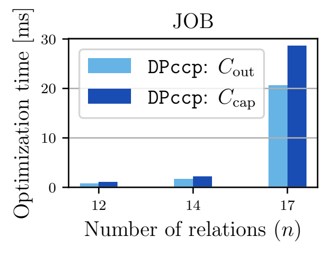

Figure 6: Initial overhead in optimizing C_{cap} on JOB

Joint Optimization: $C_{\mathrm{cap}} = C_{\mathsf{max}} + C_{\mathrm{out}}$

(Very) Simple Solution

- Replace `DPccp` in C_{max} optimization by `DPconv`.

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- Example: The optimal plan has $C_{\mathrm{out}} = 400K$ and $\varepsilon = 1\%$.
  $\rightarrow$ Then we obtain a plan of cost $\leq 404K$.
- Running time: $O^*(2^n \log W/\varepsilon)$.

Approximate $(\min, +)$ Subset Convolution

Definition

Given two set functions f, g, approximate their $(\min, +)$ subset convolution:

$$(f \star g)(S) \leq \widetilde{h}(S) \leq (1 + \varepsilon)(f \star g)(S)$$

for all $S \subseteq [n]$, with $\varepsilon > 0$.

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How to employ it

- Fix a relative error $\delta > 0$.
- Hence, the total error is $(1 + \delta)^{n-1}$.
- To obtain $(1 + \varepsilon)$-approximation, simply set $\delta := \Theta(\varepsilon/(n-1))$.

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Benchmarks

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Competitors

- `DPsub[max, out]`: $\Theta(3^n)$ – independent of the query shape.
- `DPccp` = `DPsub` on clique queries.
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- `DPsub[max, out]`: $\Theta(3^n)$ – independent of the query shape.
- `DPccp` $=$ `DPsub` on clique queries.
- A^*`[out]`: $O(\#\texttt{ccp})$ – adapts to the cardinalities.
- `DPconv[max]`: $\Theta(2^n n^3)$ – independent of the query shape.

Evaluation: C_{max}

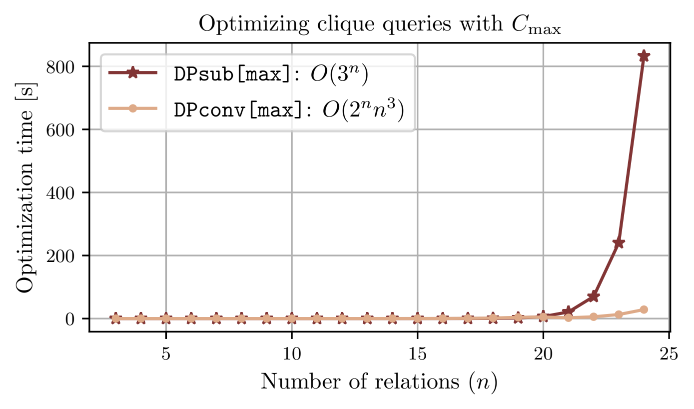

Figure 7: Optimizing for C_{max} on clique queries

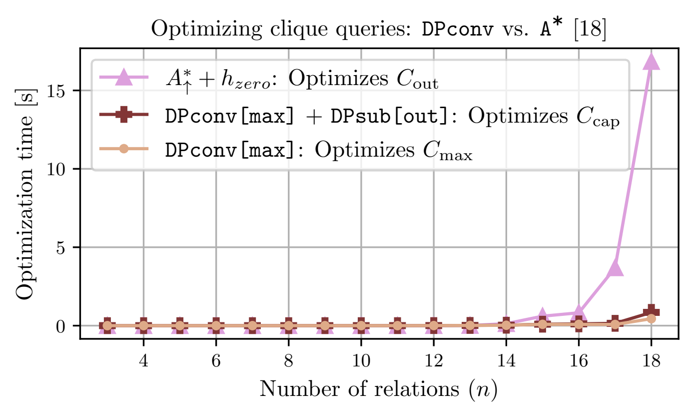

Figure 8: Optimizing for C_{out} and C_{cap} on clique queries

Evaluation: $C_{\text{cap}} = C_{\mathsf{max}} + C_{\text{out}}$

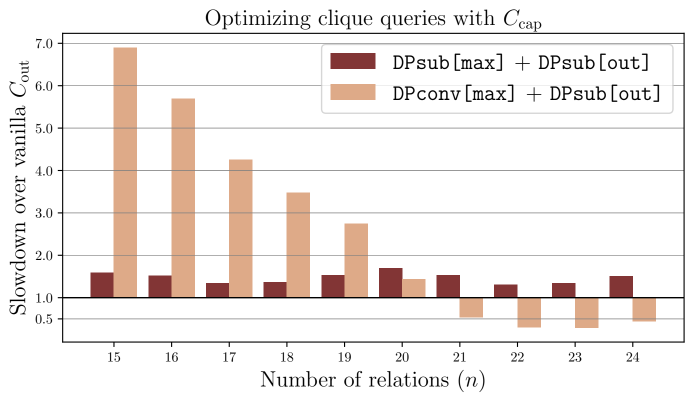

Figure 9: Optimizing for C_{cap} on clique queries

Open Problems

- Currently, `DPconv` is agnostic to the query shape.
  - For $C_{\max}$, it *always* runs in $O(2^n n^3)$-time.
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- Polynomial-*space* join ordering.
  - All exact join order optimizers take exponential *space*.
  - Example: Steiner tree can be solved in polynomial space.
- Jointly optimize the memory of concurrent queries using $C_{\max}$.
  - AutoWLM [2] can predict query's memory requirements.

Outlook

- This summer: Internship @GSL in Barcelona with Tiemo Bang.

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Evaluation: Costs

- CEB benchmark (13,644 queries).
- 2,873 queries:
  - C_{out} has 6.8% larger C_{max}.
  - C_{max} looses 22.8% in C_{out}.
  - C_{cap} looses only 9.5% in C_{out} while maintaining optimal C_{max}.